International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 11, pp: 189-196 www.ijaem.net ISSN: 2395-5252

Interval -Valued Hesitant Fuzzy Subnearring Bonferroni Mean and Interval -Valued Hesitant Fuzzy Subnearring Weighted Bonferroni Mean

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Date of Submission: 05-12-2020 Date of Acceptance: 20-12-2020

ABSTRACT: In this paper the concept of hesitant fuzzy set is introduced in the abstract Mathematical notion of nearring for further development of hesitant fuzzy set on a theoretical model. An attempt has been made to study the algebraic nature of interval-valued hesitant fuzzy subnearringbonferroni mean and interval valued hesitant fuzzy subnearring weighted Bonferroni mean of a nearring.

KEYWORDS: Hesitant Fuzzy Set, Interval-Valued Hesitant Fuzzy set, Interval-Valued Hesitant Fuzzy subset, Interval- Valued Hesitant Fuzzy Subnearring, Interval valued Hesitant Fuzzy Subnearring Bonferroni Mean, Interval Valued Hesitant Fuzzy Subnearring Weighted Bonferroni Mean.

I. INTRODUCTION

Bonferroni proposed a typical class of average mean, which is called Bonferroni Mean. It has aroused a lot of retrieval experts in latest years. A series of pervasive Bonferroni mean operators were established by Yager. It has been latter compiled by other retrieval investigators. The Bonferroni mean operators are aggregated with the intuitionistic fuzzy sample. A series of widespread intuitionistic fuzzy bonferroni mean operators were incorporated by Xia. The bonferroni mean operators are enhanced to an interval- valued intuitionistic fuzzy surroundings. A series of reluctant fuzzy bonferroni mean operators were established by Zhu and he portrayed a few geometric reluctant fuzzy bonferroni mean operators. The IVHF sample is a appropriate technique for handling the scenario where the scenario happens a absence of intellectual or inadequate messages are commonly experienced in the detailed analysis problems. The discussion makers desire to demonstrate their alternative with different interval numbers within [0 1]. The fuzzy sample, linguistic fuzzy sample, intuitionistic fuzzy sample or hesitant fuzzy sample are not manipulating the previous aspect.

1.1 Definition:
Consider $\tilde{h}_j^{\sigma(\kappa)} = \cup_{\tilde{\gamma}_j \in \tilde{h}_j}\{\tilde{\gamma}_j^{\sigma(k)L}, \tilde{\gamma}_j^{\sigma(k)U}\}$ (j=1,2….n) is a group of Interval valued hesitant fuzzy elements. If

$$\text{IVHFSNBM}\ (\tilde{h}_1^{\sigma(\kappa)}, \tilde{h}_2^{\sigma(\kappa)}, \tilde{h}_3^{\sigma(\kappa)} \ldots.. \tilde{h}_n^{\sigma(\kappa)}) = \left(\frac{1}{n(n+1)}\sum_{i,j=1, i\neq j}^{n} \tilde{h}_i^{\sigma(\kappa)a} \otimes \tilde{h}_j^{\sigma(\kappa)b}\right)^{\frac{1}{a+b}}$$

Then IVHFSNBM is called the Interval Valued Hesitant Fuzzy Subnearring Bonferroni Mean operator.

1.2Definition:
Consider $\tilde{h}_j^{\sigma(\kappa)} = \cup_{\tilde{\gamma}_j \in \tilde{h}_j}\{\tilde{\gamma}_j^{\sigma(k)L}, \tilde{\gamma}_j^{\sigma(k)U}\}$ (j=1,2….n) is a group of Interval valued hesitant fuzzy elements, $w = (w_1, w_2 \ldots \ldots .. w_n)^T$ is the weight vector of $\tilde{h}_j^{\sigma(\kappa)}$(j=1,2…n), satisfying $w_i > 0 (i = 1,2 \ldots n), \sum_{i=1}^{n} w_i = 1$. If IVHFSNWBM

$(\tilde{h}_1^{\sigma(\kappa)}, \tilde{h}_2^{\sigma(\kappa)}, \tilde{h}_3^{\sigma(\kappa)} \ldots\ldots \tilde{h}_n^{\sigma(\kappa)}) = \left(\frac{1}{n(n+1)} \sum_{i,j=1, i\neq j}^{n} (w_i \tilde{h}_i^{\sigma(\kappa)})^a \otimes (w_i \tilde{h}_i^{\sigma(\kappa)})^a\right)^{\frac{1}{a+b}}$

Then IVHFSNWBM is called the Interval Valued Hesitant Fuzzy Subnearring Weighted Bonferroni Mean operator.

II. PROPERTIES:

2.1 Theorem: Get a, b > 0 and $\tilde{h}_j^{\sigma(\kappa)} = \bigcup_{\tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}} \{[\tilde{\gamma}_j^{\sigma(k)L}, \tilde{\gamma}_j^{\sigma(k)U}]\}$ (j= 1,2…n) be a group of Interval- valued hesitant fuzzy subnearring elements. Using Interval - valued hesitant fuzzy subnearring bonferroni mean operator , the aggregated Interval valued hesitant fuzzy subnearring element is obtained as follows.

$$\mathrm{IVHFSNBM}(\tilde{h}_1^{\sigma(\kappa)}, \tilde{h}_2^{\sigma(\kappa)}, \tilde{h}_3^{\sigma(\kappa)} \ldots\ldots \tilde{h}_n^{\sigma(\kappa)}) = \bigcup \tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}, \tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}$$

$$\left\{\left[\left(1 - \prod_{\substack{i=1,j=1\\ i\neq j}}^{n} \left((1 - (\tilde{\gamma}_i^{\sigma(k)L})^a (\tilde{\gamma}_j^{\sigma(k)L})^b)^{\frac{1}{n(n+1)}}\right)\right)^{\frac{1}{a+b}}, \left(1 - \prod_{\substack{i=1,j=1\\ i\neq j}}^{n} \left((1 - (\tilde{\gamma}_i^{\sigma(k)U})^a (\tilde{\gamma}_j^{\sigma(k)U})^b)^{\frac{1}{n(n+1)}}\right)\right)^{\frac{1}{a+b}}\right]\right\}$$

Proof:

$$\tilde{h}_i^{\sigma(\kappa)a} = \bigcup_{\tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}} \{[(\tilde{\gamma}_i^{\sigma(k)L})^a, (\tilde{\gamma}_i^{\sigma(k)U})^a]\}$$

$$\tilde{h}_i^{\sigma(\kappa)b} = \bigcup_{\tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}} \{[(\tilde{\gamma}_j^{\sigma(k)L})^b, (\tilde{\gamma}_j^{\sigma(k)U})^b]\}$$

And

$$\tilde{h}_i^{\sigma(\kappa)a} \otimes \tilde{h}_i^{\sigma(\kappa)b} = \bigcup_{\tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}, \tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}} \{[(\tilde{\gamma}_i^{\sigma(k)L})^a (\tilde{\gamma}_j^{\sigma(k)L})^b, (\tilde{\gamma}_i^{\sigma(k)U})^a (\tilde{\gamma}_j^{\sigma(k)L})^b]\}$$

$$\sum_{\substack{i,j=1\\ i\neq j}}^{n} \tilde{h}_i^{\sigma(\kappa)a} \otimes \tilde{h}_i^{\sigma(\kappa)b}$$

$$= \bigcup_{\tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}, \tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}} \left\{\left[1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} 1 - (\tilde{\gamma}_i^{\sigma(k)L})^a (\tilde{\gamma}_j^{\sigma(k)L})^b, 1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} 1 - (\tilde{\gamma}_i^{\sigma(k)U})^a (\tilde{\gamma}_j^{\sigma(k)U})^b\right]\right\}$$

and

$$\frac{1}{n(n+1)} \sum_{\substack{i,j=1\\ i\neq j}}^{n} \tilde{h}_i^{\sigma(\kappa)a} \otimes \tilde{h}_i^{\sigma(\kappa)b}$$

$$= \bigcup_{\tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}, \tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}} \left\{\left[1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} (1 - (\tilde{\gamma}_i^{\sigma(k)L})^a (\tilde{\gamma}_j^{\sigma(k)L})^b)^{\frac{1}{n(n+1)}}, 1 - \prod_{\substack{i,j=1\\ i\neq j}}^{n} (1 - (\tilde{\gamma}_i^{\sigma(k)U})^a (\tilde{\gamma}_j^{\sigma(k)U})^b)^{\frac{1}{n(n+1)}}\right]\right\}$$

Therefore , we have

$$\mathrm{IVHFSNBM}(\tilde{h}_1^{\sigma(\kappa)}, \tilde{h}_2^{\sigma(\kappa)}, \tilde{h}_3^{\sigma(\kappa)} \ldots\ldots \tilde{h}_n^{\sigma(\kappa)}) = \bigcup \tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}, \tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}$$

$$\left\{\left[\left(1-\prod_{\substack{i=1,j=1\\i\neq j}}^{n}\left((1-(\tilde{\gamma}_i^{\sigma(k)L})^a(\tilde{\gamma}_j^{\sigma(k)L})^b)^{\frac{1}{n(n+1)}}\right)\right)^{\frac{1}{a+b}},\left(1-\prod_{\substack{i=1,j=1\\i\neq j}}^{n}\left((1-(\tilde{\gamma}_i^{\sigma(k)U})^a(\tilde{\gamma}_j^{\sigma(k)U})^b)^{\frac{1}{n(n+1)}}\right)\right)^{\frac{1}{a+b}}\right]\right\}$$

Hence the proof.

2.2 Theorem: Get $\tilde{h}_j^{\sigma(\kappa)} = \cup_{\tilde{\gamma}_j^{\sigma(k)}\in\tilde{h}_j^{\sigma(\kappa)}}\{[\tilde{\gamma}_j^{\sigma(k)L},\tilde{\gamma}_j^{\sigma(k)U}]\}$ (j= 1,2…n) be a group of Interval- valued hesitant fuzzy subnearring elements, $w = (w_1, w_2 \dots\dots w_n)^T$ is the weight vector of $\tilde{h}_j^{\sigma(\kappa)}(j = 1,2 \dots n)$, satisfying $w_i > 0(i = 1,2 \dots n)$, and $\sum_{i=1}^{n} w_i = 1$. Then, the IVHFSNBM operator can be transformed as follows.

IVHFSNWBM$(\tilde{h}_1^{\sigma(\kappa)},\tilde{h}_2^{\sigma(\kappa)},\tilde{h}_3^{\sigma(\kappa)} \dots\dots \tilde{h}_n^{\sigma(\kappa)}) = \cup\, \tilde{\gamma}_i^{\sigma(k)} \in \tilde{h}_i^{\sigma(\kappa)}, \tilde{\gamma}_j^{\sigma(k)} \in \tilde{h}_j^{\sigma(\kappa)}$

$$\left\{\left[\left(1-\prod_{\substack{i=1,j=1\\i\neq j}}^{n}\left((1-(1-(1-\tilde{\gamma}_i^{\sigma(k)L})^{w_i})^a(1-(1-\tilde{\gamma}_i^{\sigma(k)L})^{w_j})^b)^{\frac{1}{n(n+1)}}\right)\right)^{\frac{1}{a+b}},\left(1-\prod_{\substack{i=1,j=1\\i\neq j}}^{n}\left((1-(1-(1-\tilde{\gamma}_i^{\sigma(k)U})^{w_i})^a(1-(1-\tilde{\gamma}_i^{\sigma(k)U})^{w_j})^b)^{\frac{1}{n(n+1)}}\right)\right)^{\frac{1}{a+b}}\right]\right\}$$

III. REDESIGNATION OF FACULTY EVALUATION PROBLEM IN DEEMED UNIVERSITY

Here we illustrate a new approach with practical example on redesignation of the faculty in deemed universities.

The general designations are 1 Professor 2.Associate Professor 3. Senior Assistant Professor 4. Assistant Professor 5. Lecturer. Assume that there are five designations that need to be evaluated (v_1, v_2, v_3, v_4, v_5). According to the assessment guidelines, a group of three experts (e_1, e_2, e_3) take responsibility for this assessment.

When evaluating the five designations, the three experts mainly consider the following four aspects and the weight vector is assigned as (0.1, 02, 0.2, 0.1) according to the guidelines set out by the UGC.

u_1 = API (Academic Performance Index) (Years of Teaching Experience)
u_2 = Publication (Scopus / EBSCO/ Pubmed)
u_3 = Research Objectives (Ph.d/ Ph.dPersuing)
u_4 = Funding Projects / Patents

The three experts are Academic Dean , Academic Director and Registrar

S.No	Norms	Description of Evaluation	Results	Minima	Maxima
1	API (Academic Performance Index) (Years of Teaching Experience) (u_1)	The Minima or Maxima of the surrogate v_i meets the norm u_1	v_1 v_2 v_3 v_4 v_5	0.2 0.3 0.2 0.4 0.1	0.6 0.5 0.5 0.7 0.3
2	Publication (Scopus / EBSCO/ Pubmed) (u_2)	The Minima or Maxima of the surrogate v_i meets the norm u_2	v_1 v_2 v_3 v_4 v_5	0.5 0.5 0.4 0.2 0.4	0.7 0.6 0.6 0.3 0.5
3	Research Objectives (Ph.d/ Ph.dPersuing) (u_3)	The Minima or Maxima of the	v_1 v_2	0.5 0.1	0.7 0.3

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 11, pp: 189-196 www.ijaem.net ISSN: 2395-5252

		surrogate v_i meets the norm u_3	v_3 v_4 v_5	0.9 0.3 0.2	0.2 0.5 0.4
4	Funding Projects / Patents (u_4)	The Minima or Maxima of the surrogate v_i meets the norm u_4	v_1 v_2 v_3 v_4 v_5	0.4 0.3 0.5 0.2 0.4	0.8 0.4 0.6 0.4 0.5

Table 1. Scrutiny table for Academic Dean

S.No	Norms	Description of Evaluation	Results	Minima	Maxima
1	API (Academic Performance Index) (Years of Teaching Experience) (u_1)	The Minima or Maxima of the surrogate v_i meets the norm u_1	v_1 v_2 v_3 v_4 v_5	0.2 0.5 0.2 0.4 0.1	0.6 0.7 0.5 0.5 0.6
2	Publication (Scopus / EBSCO/ Pubmed) (u_2)	The Minima or Maxima of the surrogate v_i meets the norm u_2	v_1 v_2 v_3 v_4 v_5	0.3 0.5 0.5 0.5 0.4	0.5 0.6 0.8 0.6 0.5
3	Research Objectives (Ph.d/ Ph.dPersuing) (u_3)	The Minima or Maxima of the surrogate v_i meets the norm u_3	v_1 v_2 v_3 v_4 v_5	0.5 0.1 0.3 0.3 0.2	0.7 0.3 0.4 0.5 0.4
4	Funding Projects / Patents (u_4)	The Minima or Maxima of the surrogate v_i meets the norm u_4	v_1 v_2 v_3 v_4 v_5	0.4 0.4 0.5 0.2 0.4	0.8 0.8 0.6 0.4 0.5

Table 2. Scrutiny table for Academic Director

S.No	Norms	Description of Evaluation	Results	Minima	Maxima
1	API (Academic Performance Index) (Years of Teaching Experience) (u_1)	The Minima or Maxima of the surrogate v_i meets the norm u_1	v_1 v_2 v_3 v_4 v_5	0.3 0.4 0.4 0.3 0.1	0.5 0.6 0.7 0.6 0.3
2	Publication (Scopus / EBSCO/ Pubmed) (u_2)	The Minima or Maxima of the surrogate v_i meets the norm u_2	v_1 v_2 v_3 v_4 v_5	0.4 0.5 0.5 0.5 0.2	0.5 0.6 0.6 0.6 0.5
3	Research Objectives (Ph.d/ Ph.dPersuing) (u_3)	The Minima or Maxima of the surrogate v_i meets the norm u_3	v_1 v_2 v_3 v_4	0.5 0.4 0.3 0.3	0.7 0.5 0.4 0.5

			v_5	0.2	0.4
4	Funding Projects / Patents (u_4)	The Minima or Maxima of the surrogate v_i meets the norm u_4	v_1 v_2 v_3 v_4 v_5	0.4 0.4 0.5 0.3 0.2	0.8 0.8 0.6 0.3 0.5

Table 3. Scrutiny table for Registrar

The overall report as follows

Surrogates	u_1	u_2	u_3	u_4
v_1	{[0.2,0.6],[0.3,0.5]}	{[0.5,0.7],[0.3,0.5],[0.4,0.5] }	{[0.5,0.7]}	{[0.4,0.8] }
v_2	{[0.3,0.5],[0.5,0.8],[0.4,0.6]}	{[0.5,0.6]}	{[0.1,0.3],[0. 4,0.5]}	{[0.3,0.4], [0.4,0.8]}
v_3	{[0.2,0.5],[0.4,0.7]}	{[0.4,0.6],[0.5,0.8],[0.5,0.6] }	{[0.9,0.2],[0. 3,0.4]}	{[0.5,0.6] }
v_4	{[0.4,0.7],[0.4,0.5],0.3,0.6] }	{[0.2,0.3],[0.5,0.6]}	{[0.3,0.5]}	{[0.2,0.4], [0.3,0.3]}
v_5	{[0.1,0.3],[0.1,0.6]}	{[0.4,0.5],[0.2,0.5]}	{[0.2,0.4]}	{[0.4,0.5], [0.2,0.5]}

Table 4. Interval Valued Hesitant Fuzzy Decision Matrix

Parameters	v_1	v_2	v_3	v_4	v_5	**Ranking**
a=0.0001, b=15	0.900087612	0.900085181	0.900081026	0.800172118	0.700242207	$v_1 > v_2 > v_3 > v_4 > v_5$
a = b = 10	0.519615242	0.424264068	0.3	0.489897948	0.374165738	$v_1 > v_4 > v_2 > v_5 > v_3$
a = b = 15	0.51995552	0.424627988	0.300361409	0.490247644	0.374533745	$v_1 > v_4 > v_2 > v_5 > v_3$

Table 5. Ranking alternatives with the help of IVHFSNWBM operator

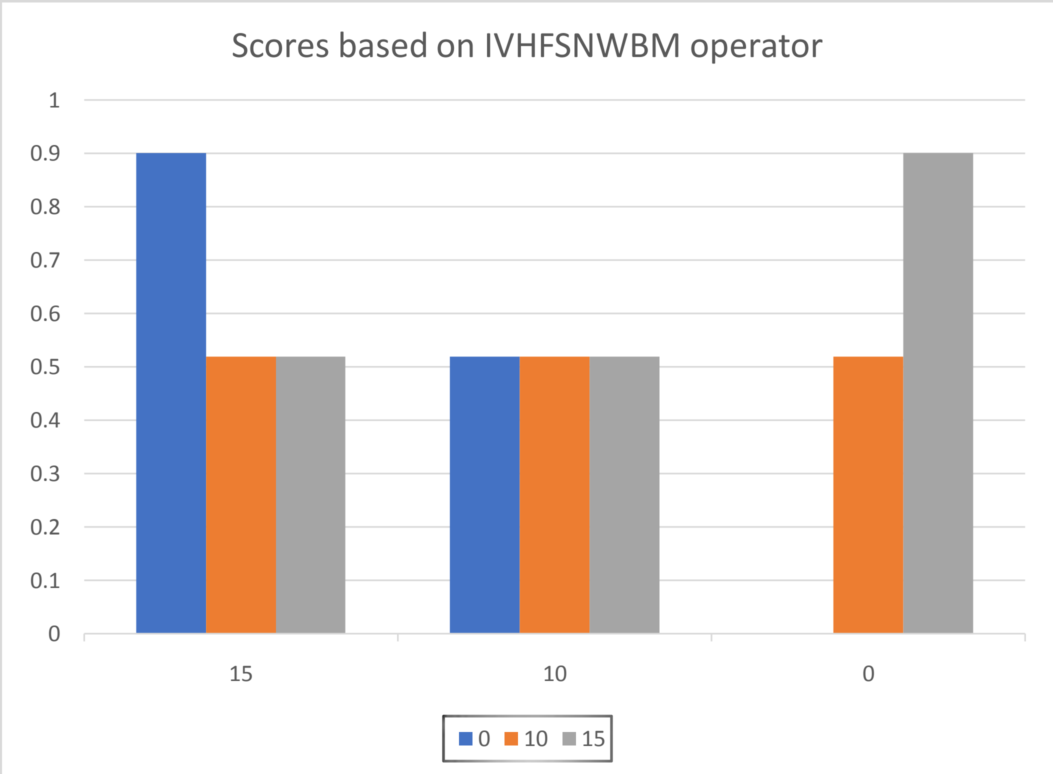

Figure 1: Score for alternative v_1 obtained by the IVHFSNWBM operator

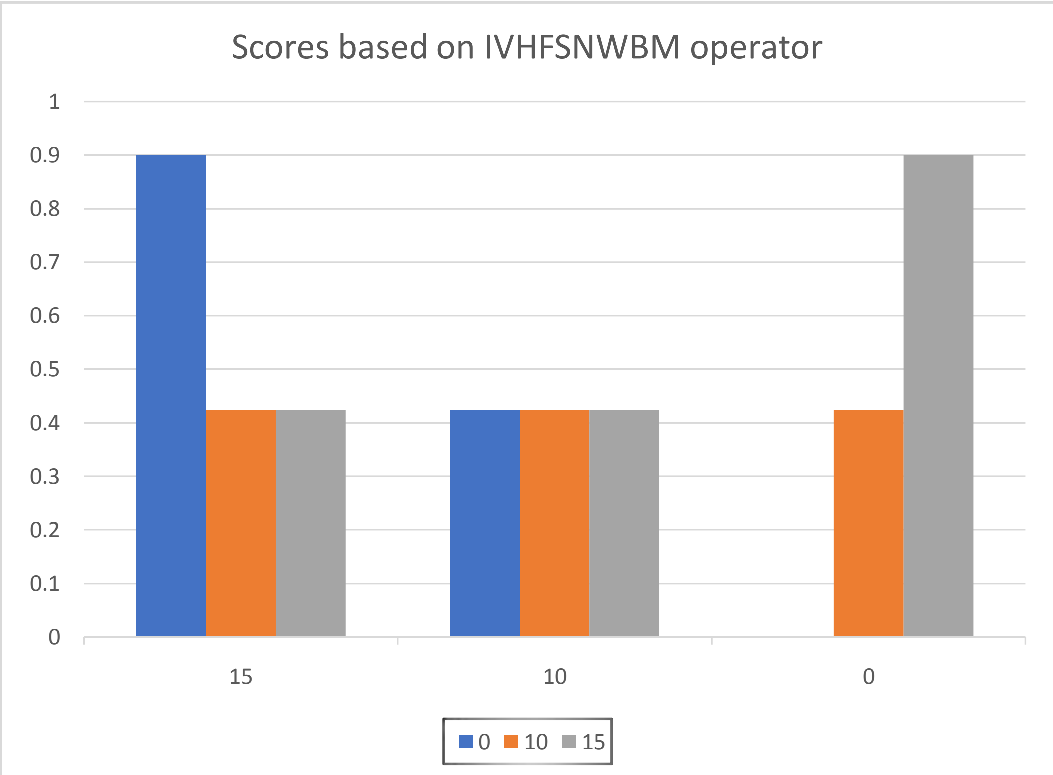

Figure 2: Score for alternative v_2 obtained by the IVHFSNWBM operator

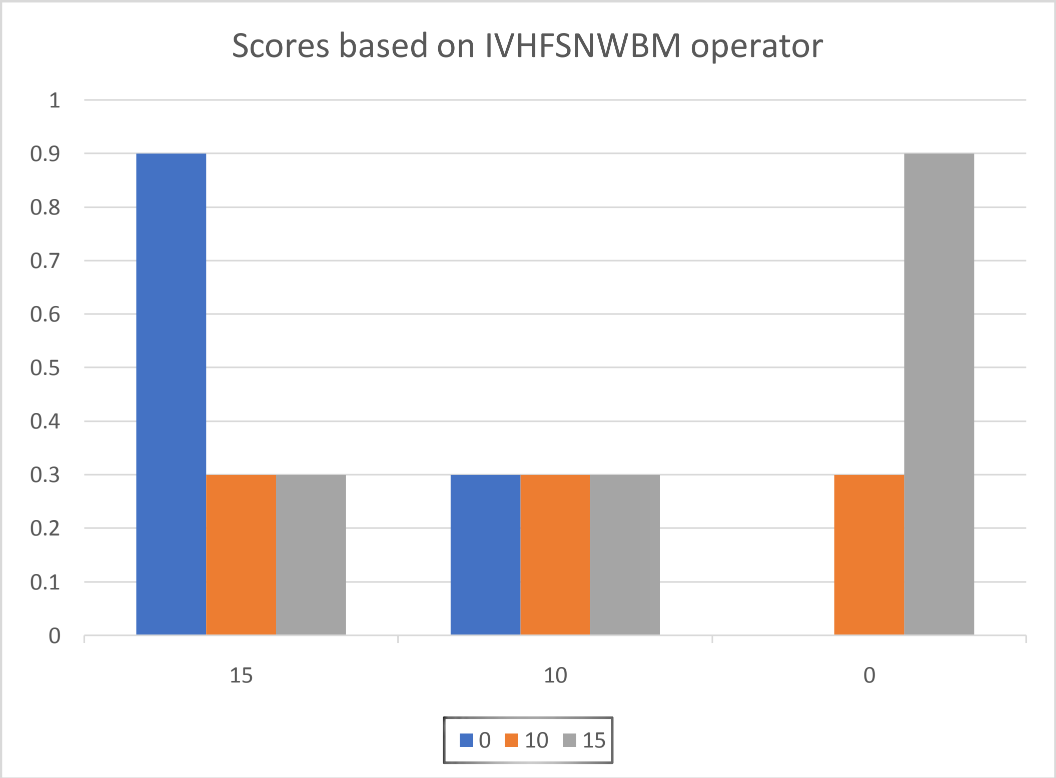

Figure 3: Score for alternative v_3 obtained by the IVHFSNWBM operator

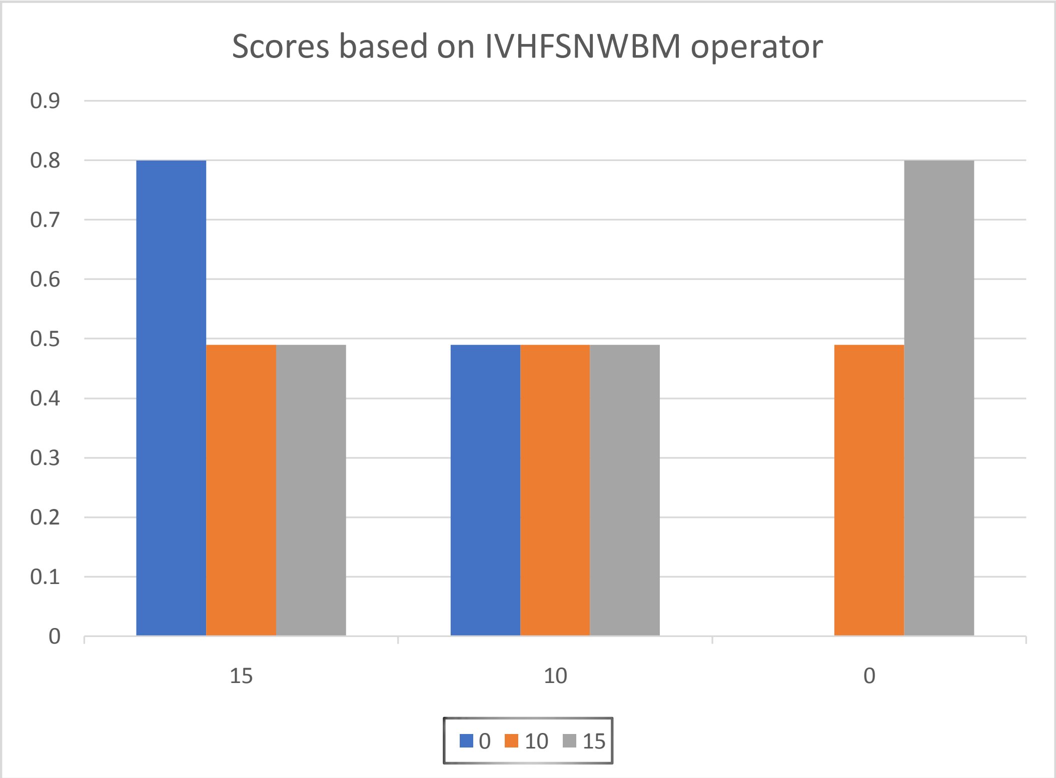

Figure 4: Score for alternative v_4 obtained by the IVHFSNWBM operator

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 11, pp: 189-196 www.ijaem.net ISSN: 2395-5252

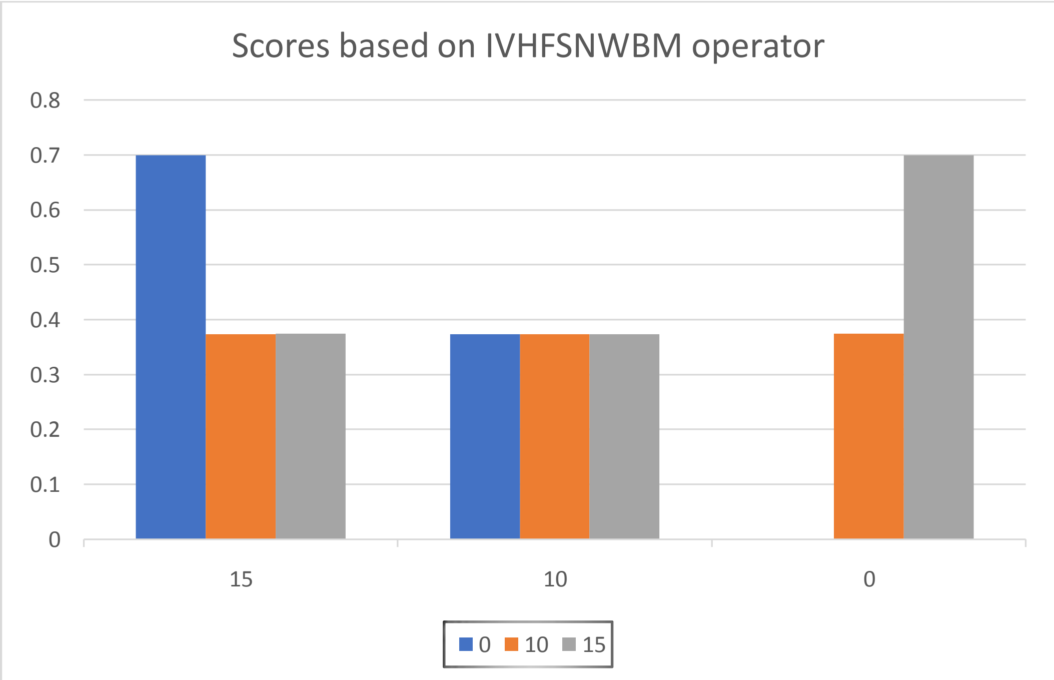

Figure 5: Score for alternative v_5 obtained by the IVHFSNWBM operator

Comparison between the proposed method with the other method which is proposed by Chen N, Xu Z and Xia M as given below.
The score functions of each alternative based on Interval – valued hesitant fuzzy weighted arithmetic operator are computed and shown below.
$S_1 = 0.1266$; $S_2 = 0.1885$; $S_3 = 0.1840$; $S_4 = 0.1546$; $S_5 = 0.1325$
Based on these, the rank of the alternatives are
$v_2 > v_3 > v_4 > v_5 > v_1$
The best alternative is v_2. That is the Interval – valued hesitant fuzzy weighted arithmetic operator does not consider the correlation between the aggregative arguments which are not perfect.

IV. CONCLUSION

This paper is concluded that, theinterval -valued hesitant fuzzy subnearringbonferroni mean and interval -valued hesitant fuzzy subnearring weighted bonferroni mean are existed.

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